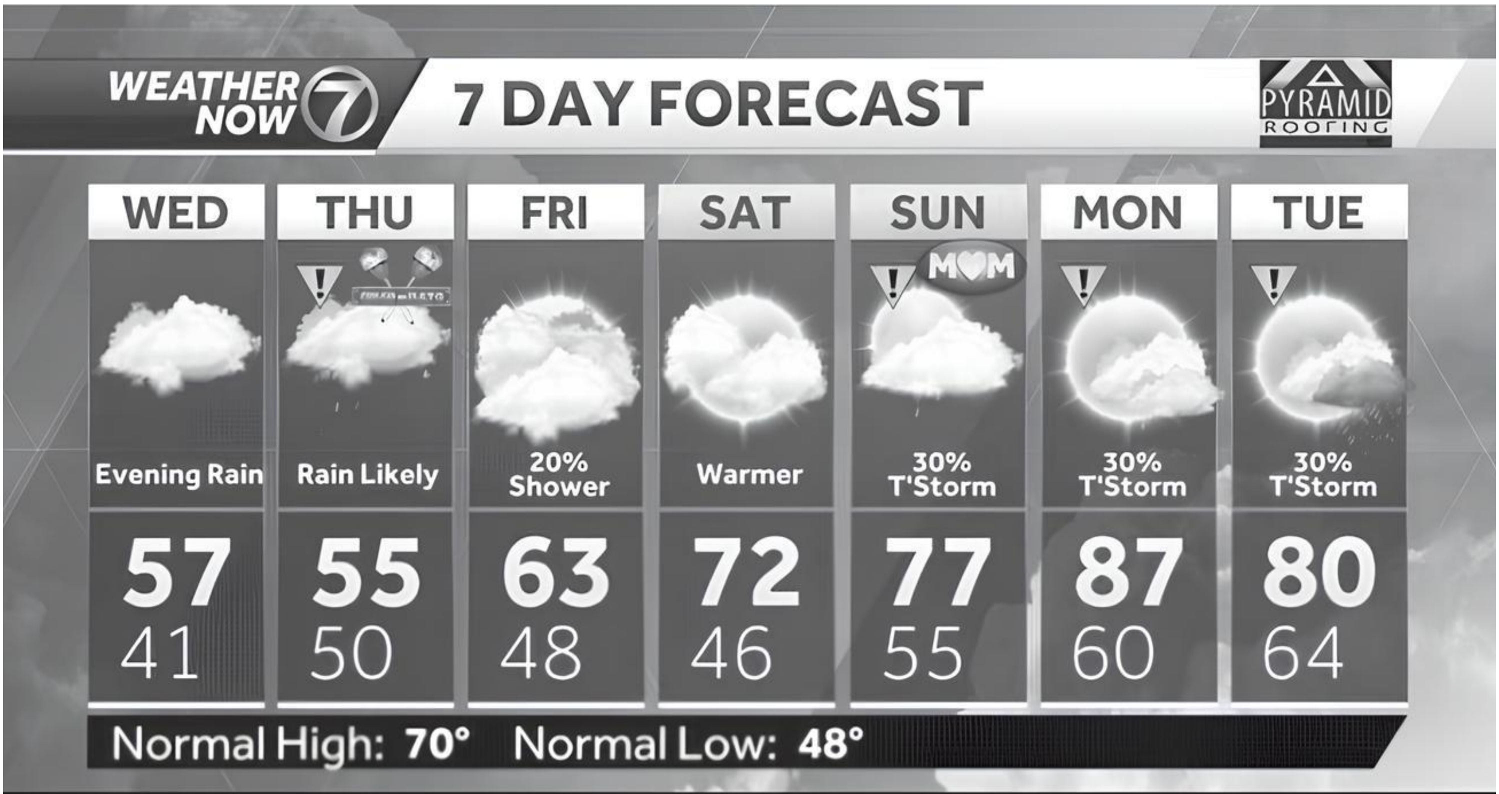
14 Probability



A probability forecast is a percentage-based evaluation of the likelihood of an occurrence occurring, as well as a record of the hazards connected with weather. Meteorologists utilize a variety of devices and technologies to forecast weather changes throughout the world. They compile a global weather prediction database to anticipate temperature changes and likely weather conditions for a certain hour, day, week, and month.

Topic Notes

- *Event and Algebra of Events*

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EVENT AND ALGEBRA OF EVENTS 1

TOPIC 1

EVENT

A subset of the sample space associated with a random experiment is called an event, generally denoted by 'E'.

An event associated with a random experiment is said to occur, if any one of the elementary events associated with it is an outcome of the experiment.

For eg: Suppose a die is thrown, the sample space

$S = \{1, 2, 3, 4, 5, 6\}$. Then,

$E = \{2, 3, 4\}$ is an event.

Also, if the outcome of the experiment is, 4. Then we say that event E has occurred.

Occurrence of an Event

An event associated with a random experiment is said to occur, if any one of the elementary events associated to it is an outcome of the experiment.

e.g. suppose a die is thrown and let A be an event of getting an even number.

Then, $A = \{2, 4, 6\}$

Types of Events

On the basis of the element in an event, events are classified into the following types:

Simple Event

If an event has only one sample point of the sample space, it is called a simple (element) event.

E.g., Let a die is thrown, then sample space,

$S = \{1,2,3,4,5,6\}$

Then, $A = \{4\}$ and $B = \{6\}$ are simple events.

Compound Event

If an event has more than one sample point of the sample space, then it is called a compound event.

E.g., on rolling a die, we have the sample space,

$S = \{1, 2, 3, 4, 5, 6\}$

Then, $E = \{2, 4, 6\}$, $F = \{1, 3, 5\}$, are compound events.

Sure Event

The event which is certain to occur is said to be the sure event. The whole sample 'S' is a sure or certain event, since it is a subset of itself.

E.g., on throwing a die, we have sample space,

$S = \{1,2,3,4,5,6\}$

Then, E = Event of getting a natural number less than 7, is a sure event, since $E = \{1, 2, 3, 4, 5, 6\} = S$

Impossible Event

The event which has no element is called an impossible event or null event. The empty set 'ϕ' is an impossible event, since it is a subset of sample space S.

E.g., on throwing a die, we have the sample space,

$S = \{1, 2, 3, 4, 5, 6\}$

Then, E = event of getting a number less than 1, is an impossible event, since $E = \phi$.

Equally Likely Events

Events are called equally likely when we do expect the happening of one event in preference to the other.

In general, events $E_1, E_2, ..., E_n$ be n subsets of a sample space S. Then, events $E_1, E_2, ..., E_n$ are exhaustive events, if $E_1 \cup E_2 \cup E_3 \cup ... \cup E_n = S$

E.g., consider the experiment of throwing a die. Then, $S = \{1, 2, 3, 4, 5, 6\}$

Let E_1 = event of getting a number less than 3.

E_2 = event of getting an odd number.

E_3 = event of getting a number greater than 3.

Then, $E_1 = \{1, 2\}$, $E_2 = \{1, 3, 5\}$, $E_3 = \{4, 5, 6\}$

Thus, $E_1 \cup E_2 \cup E_3 = S$. Hence, E_1, E_2, E_3 are exhaustive events.

Example 1.1: A die is thrown. Describe the following events:

(A) A : a number less than 7

(B) B : a number greater than 7

(C) C : a multiple of 3

(D) D : a number less than 4

(E) E : an even number greater than 4

(F) F : a number not less than 3

Ans. (A) If the die is thrown,

Possible outcomes are 1, 2, 3, 4, 5, 6

$S = \{1, 2, 3, 4, 5, 6\}$

A : a number less than 7

$A = \{1, 2, 3, 4, 5, 6\}$

(B) $S = \{1, 2, 3, 4, 5, 6\}$

There is no number greater than 7

Hence, $B = \phi$

(C) Possible multiple of 3 are 3, 6

Hence, $C = \{3,6\}$

(D) $S = \{1, 2, 3, 4, 5, 6\}$

D : a number less than 4.

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Numbers less than 4 are 1,2,3.

Hence, D = {1,2,3}

(E) S = {1,2,3,4,5,6}

E : an even number greater than 4

Even number greater than 4 is 6 only

Hence, E = {6}

(F) S = {1,2,3,4,5,6}

F : a number not less than 3

Number not less than 3 are 3, 4, 5 and 6

Hence, F = {3,4,5,6}

Example 1.2: Using the value of example 1; find

(A) $A \cup B$	(B) $B \cup C$	(C) $A \cap B$
(D) $E \cap F$	(E) $D \cap E$	(F) $D - E$
(G) $A - C$	(H) F'	(I) $E \cap F'$

Ans.

(A) $A \cup B$ $A = \{1,2,3,4,5,6\}$ $B = \phi$ $A \cup B = \{1,2,3,4,5,6\} \cup \phi$ $= \{1,2,3,4,5,6\}$	(B) $B \cup C$ $B = \phi$ $C = \{3, 6\}$ $B \cup C = \phi \cup \{3,6\}$ $= \{3, 6\}$
(C) $A \cap B$ $A = \{1, 2, 3, 4, 5, 6\}$ $B = \phi$ $A \cap B = \{1,2,3,4,5,6\} \cup \phi$ $= \phi$	(D) $E \cap F$ $E = \{6\}$ $F = \{3,4,5,6\}$ $E \cap F = \{6\} \cap \{3,4,5,6\}$ $= \{6\}$
(E) $D \cap E$ $D = \{1,2,3\}$ $E = \{6\}$ $D \cap E = \{1,2,3\} \cap \{6\}$ $= \phi$	(F) $D - E$ $D = \{1,2,3\}$ $E = \{6\}$ $D - E = \{1,2,3\} - \{6\}$ $= \{1,2,3\}$
(G) $A - C$ $A = \{1,2,3,4,5,6\}$ $C = \{3,6\}$ $A - C = \{1,2,3,4,5,6\} - \{3,6\}$ $= \{1,2,4,5\}$	(H) F' $F = \{3,4,5,6\}$ $S = \{1,2,3,4,5,6\}$ $F' = S - F$ $= \{1,2,3,4,5,6\} - \{3,4,5,6\}$ $= \{1,2\}$
(I) $E \cap F'$ $E = \{6\}$ $F' = \{1,2\}$ $E \cap F' = \{6\} \cap \{1,2\}$ $= \phi$	

Algebra of Events

Let A and B be two events associated with a sample space S, then:

Complementary Event

For every E, there corresponds another event E′ called the complementary event of E, which consists of those outcomes that do not correspond to the occurrence of E. E′ is also called the event 'not E'.

E.g., In tossing three coins, the sample space is

S = {HHH, HHT, HTH, HTT, THH, THT, TTH, TTT}

Let E = {THT, TTH, HTT} = the event of getting only one head.

Then, E′ = {HHT, HTH, THH, TTT, HHH}.

The Event A or B

The event 'A or B' is same as the event $A \cup B$ and it contains all those elements which are either in event A or in B or in both. Thus,

$A \text{ or } B = A \cup B = \{x : x \in A \text{ or } x \in B\}$

The Event A and B

The event 'A and B' is same as the event '$A \cap B$' and it contains all those elements which are both in A and B. Thus,

$A \text{ and } B = A \cup B\ \{x : x \in A \text{ and } x \in B\}$

The Event A but not B

The event A but not B is the same as the event $A - B = (A \cap B')$ and it contains all those elements which are in A but not in B.

Then, A but not in $B = A - B = \{x : x \in A \text{ or } x \notin B\}$.

Events Equivalent Sets

The following are some events and their corresponding equivalent sets.

(1) Neither A nor B

$\bar{A} \cap \bar{B}$ or $U - (A \cap B)$ [U – universal set]

(2) Exactly one of A and B

$(A \cap \bar{B}) \cup (\bar{A} \cap B)$ or $(A \cap B) - (A \cap B)$

(3) At least one of A, B or C

$A \cup B \cup C$

(4) All three of A, B and C

$A \cap B \cap C$

(5) Exactly two of A, B and C

$(A \cap B \cap \bar{C}) \cup (A \cap B \cap C) \cup (A \cap B \cap C)$

Mutually Exclusive Events

Two events are said to be mutually exclusive, if the occurrence of any one of them excludes the occurrence of the other event, i.e., they cannot occur simultaneously. Thus, two events N_1 and N_2 are said to be mutually exclusive, If $N_1 \cap N_2 = \phi$.

e.g., on throwing a die, we have the sample space S = {1, 2, 3, 4, 5, 6}

Let N_1 = Event of getting even numbers = {2, 4, 6}

And N_2 = Event of getting odd number = {1, 3, 5}

Then, $N_1 \cap N_2 = \phi$.

So, N_1 and N_2 are mutually exclusive events.

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Exhaustive Events

For a random experiment, a set of events is said to be exhaustive, if one of them necessarily occurs whenever the experiment is performed. Let $N_1, N_2, N_3, ..., N_n$ be n subsets of a sample space S. Then, events $N_1, N_2, N_3, ...,$ N_n are called exhaustive events, if $N_1 \cup N_2 \cup E_3 \cup ... \cup N_n = S$

e.g., Consider the experiment of throwing a die. We have,

S = {1, 2, 3, 4, 5, 6}. Let us define the following events

N_1 : A number less than 4 appears.

N_2 : A number greater than 2 but less than 5 appears.

N_3 : A number greater than 4 appears.

Then, $E_1 = \{1, 2, 3\}, E_2, = \{3, 4\}, E_3 = \{5, 6\}$

We observe that

$E_1 \cup E_2 \cup E_3 = \{1, 2, 3\} \cup \{3, 4\} \cup \{5, 6\} = S$

Thus, the events E_1, E_2 and E_3 are exhaustive events.

Important

- *A sample spaces is called a discrete sample space, if S is a finite set.*
- *We can define as many events as these are subsets of a sample space. Thus, the number of events of a sample space S is 2^n, where 'n' is the number of elements in S.*
- *Elementary events associated with a random experiments are also known as in decomposable events.*
- *All events other than elementary events and impossible events associated with a random experiment are called compound events.*
- *For any event E, associated with a sample space S, E' = not E = S – E = {ω: ω ∈ S and ω ∉ E}.*
- *Simple events of a sample space are always mutually exclusive.*
- *If $E_i \cap E_j = \phi$ for, i ≠ j i.e., events E_i and E_j are pair wise disjoint and $E_1 \cup E_2 \cup ... \cup E_n = S$, then events $E_1, E_2, ..., E_n$ are called mutually exclusive and exhaustive events.*

TOPIC 2

AXIOMATIC APPROACH TO PROBABILITY

Probabilities of Equally Likely Outcomes

All the outcomes are said to be equal if the chances of occurrence of each simple event is the same.

Let S be the sample space of an experiment

$S = \{e_1, e_2, e_3, e_4, ..., e_n\}$

i.e., $P(e_1) = \phi$ for all $e_1 \in S$ where $0 \leq PS$.

Since, $\sum_i^n = 1 \ P(e_1) = 1$

i.e., $p + p + p ... p$ ($n$times) = 1

Or $np = 1$ i.e., $-p = \frac{1}{n}$

Example 1.3: A coin is tossed twice, what is the probability that at least one tail occurs?

Ans. When 2 coins are tossed,

Sample Space = S = {HH, HT, TH, HT}

$n(S) = 4$

Let A be the event that at least 1 tail occurs

Hence A = {HT, TH, TT}

$n(A) = 3$

$$P(A) = \frac{\text{Number of outcomes favourable to A}}{\text{Total number of possible outcomes}}$$

$$= \frac{n(A)}{n(S)}$$

$$= \frac{3}{4}$$

Probability of the Event 'A' or B' / Addition Rule of Probability

If N and M are two events associated with a random experiment, then

$P(N \cup M) = P(N) + P(M) - P(N \cap M)$

i.e., P(N or M) = P(N) + P(M) – P(N and M)

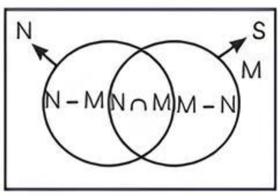
It is known as the additional law of probability for two events.

Also,

(i) when events N and M are mutually exclusive, then $P(N \cup M) = P(N) + P(M)$

(ii) when events N and M are mutually exclusive and exhaustive, then $P(N \cup M) = P(N) + P(M) = 1$

e.g., If $P(N) = \frac{4}{5}$ and $P(M) = \frac{7}{15}$



Example 1.4: If E and F are events such that $P(E) = \frac{7}{15}$, $P(F) = \frac{1}{2}$ and P(E and F) = $\frac{1}{8}$, find:

(A) P(E or F) **(B)** P(not E and not F)

Ans. (A) $P(E \text{ and } F) = P(E \cap F) = \frac{1}{8}$

We need to find

$P(E \text{ or } F) = P(E \cup F)$

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We know that

$$P(E \cup F) = P(E) + P(F) - P(E \cap F)$$

Putting values

$$P(E \cup F) = \frac{1}{4}+\frac{1}{2}-\frac{1}{8}$$

$$= \frac{2+4-1}{8}$$

$$= \frac{6-1}{8}$$

$$= \frac{5}{8}$$

(B) P(not E and not F)

$$= P(E' \cap F')$$

$$= P(E \cup F)'$$

$$= 1 - P(E \cup F)$$

[By Demorgan's law]

$$= 1 - \frac{5}{8}$$

$$= \frac{8-5}{8}$$

$$= \frac{3}{8}$$

Probability of Event 'not A'

A is the event of drawing card from a deck of ten cards numbered from 1 to 10.

Let S be the sample space, S = {1,2,3, ..., 10}

If all the outcomes 1, 2, ..., 10 are considered to be equally likely, then the probability of each outcome is $\frac{1}{10}$.

Now, P(A) = P(2) + P(4) + P(6) + P(8)

$$= \frac{1}{10}+\frac{1}{10}+\frac{1}{10}+\frac{1}{10}$$

$$= \frac{4}{10}=\frac{2}{5}$$

Also event 'not A' = A' = {1, 3, 5, 7, 9, 10}

Now, P(A') = P(1) + P(3) + P(5) + P(7) + P(9) + P(10)

$$= \frac{6}{10}=\frac{3}{5}$$

Thus, $P(A') = \frac{3}{5}$

Also, $P(A') = 1 - P(A) = 1-\frac{2}{5} = \frac{3}{5}$

Important

➥ *N and N' are mutually exclusive and exhaustive events.*

i.e. $N \cap N' = \phi$ *and* $N \cup N' = S$

or $P(N \cup N') = P(S)$

$P(N) + P(N') = 1$

$P(N') = P(\text{not } N) = 1 - P(N)$

Example 1.5: Three coins are tossed once. Find the probability of getting:

(A) 3 tails (B) exactly two tails

(C) no tails (D) at most two tails

Ans. (A) 3 tails

Let F be the event of getting 3 tails.

F = {TTT}

$n(F) = 1$

Probability of getting 3 tails = P(F)

$$= \frac{n(F)}{n(S)}$$

$$= \frac{1}{8}$$

(B) exactly two tails

Let G be the event of getting exactly 2 tails

G = {HTT, THT, TTH}

$n(G) = 3$

Probability of getting exactly two tails = P(G)

$$= \frac{n(G)}{n(S)}$$

$$= \frac{3}{8}$$

(C) no tails

Let H be the event of getting no tail

H = {HHH}

n(H) = 1

Probability of getting no tails = P(H)

$$= \frac{n(H)}{n(S)}$$

$$= \frac{1}{8}$$

(D) at most two tails

Let I be the event of getting at most 2 tails.

i.e., getting 0 tails, 1 tail or 2 tail

I = {HHH, HHT, HTH, THH, HTT, THT, TTH}

$n(I) = 7$

Probability of getting at most tails = P(I)

$$= \frac{n(I)}{n(S)}$$

$$= \frac{7}{8}$$

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Example 1.6: From the employees of a company, 5 persons are selected to represent them in the managing committee of the company. Particulars of five persons are as follows:

S. No	Name	Sex	Age in years
1.	Harish	M	30
2.	Rohan	M	33
3.	Sheetal	F	46
4.	Alis	F	28
5.	Salim	M	41

A person is selected at random from this group to act as a spokesperson. What is the probability that the spokesperson will be either male or over 35 years?

Ans. Total number of persons = 5

So, $n(S) = 5$

Probability spokesperson is male

There are 3 males

So, $n(A) = 5$

Probability spokesperson is male = P(A)

$$= \frac{n(A)}{n(S)}$$

$$= \frac{3}{5}$$

Probability spokesperson is over 35 years old

Let B be the event that person selected is over 35

There are 2 person over 35 years old.

So, $n(B) = 2$

Probability that the spokesperson is over 35 years old = P(B)

$$= \frac{n(B)}{n(S)}$$

$$= \frac{2}{5}$$

We need to find probability that the spokesperson will be either male or over 35 years = $P(A \cup B)$

We know that

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Probability that spokesperson is male and over 35 years old

Here, 1 person is both male and over 35 years old.

So, $n(A \cap B) = 1$

$$P(A \cap B) = \frac{n(A \cap B)}{n(S)}$$

$$= \frac{1}{5}$$

Now,

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= \frac{3}{5} + \frac{2}{5} - \frac{1}{5}$$

$$= \frac{3+2-1}{5}$$

$$= \frac{4}{5}$$

Hence, probability that the spokesperson will be either male or over 35 years

$$= P(A \cup B)$$

$$= \frac{4}{5}$$

Example 1.7: Check whether the following probabilities are consistently defined.

(A) $P(A) = 0.5, P(B) = 0.7, P(A \cap B) = 0.6$

(B) $P(A) = 0.5, P(B) = 0.4, P(A \cup B) = 0.8$

Ans. (A) To be consistently defined $P(A \cap B)$ should be less than or equal to P(A) and also P(B) but here $P(A \cap B) > P(A)$ {Here, $0.6 > 0.5$}. Hence, not consistent.

(B) Using, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$P(A \cap B) = P(A) + P(B) - P(A \cup B)$$

$$\Rightarrow P(A \cap B) = 0.5 + 0.4 - 0.8$$

= 0.1, which is true since it is less than P(A) and P(B)

$\therefore$ The given probabilities are consistently defined.

Example 1.8: Fill in the blanks in following table:

	P(A)	P(B)	$P(A \cap B)$	$P(A \cup B)$
(A)	$\frac{1}{3}$	$\frac{1}{5}$	$\frac{1}{15}$	?
(B)	0.35	?	0.25	0.6
(C)	0.5	0.35	?	0.7

Ans. (A) Here, $P(A) = \frac{1}{3}$, $P(B) = \frac{1}{5}$, $P(A \cap B) = \frac{1}{15}$

We know,

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= \frac{1}{3} + \frac{1}{5} - \frac{1}{15}$$

$$= \frac{8}{15} - \frac{1}{15}$$

$$= \frac{7}{15}$$

(B) Here, $P(A) = 0.35$, $P(B) = ?$, $P(A \cap B) = 0.25$, $P(A \cup B) = 0.6$

We know,

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

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$\Rightarrow \quad P(B) = P(A \cup B) + P(A \cap B) - P(A)$

$= 0.6 + 0.25 - 0.35$

$= 0.85 - 0.35$

$= 0.5$

(C) Here, $P(A) = 0.5$, $P(B) = 0.35$, $P(A \cup B) = 0.7$

We know,

$P(A) + P(B) - P(A \cap B) = P(A \cup B)$

$\Rightarrow \quad P(A \cap B) = P(A) + P(B) - P(A \cup B)$

$= 0.5 + 0.35 - 0.7 = 0.15$

Example 1.9: If E and F are events such that $P(E') = \frac{1}{4}$, $P(F') = \frac{1}{2}$ and $P(E \text{ and } F') = \frac{1}{8}$, find

(A) P(E or F') **(B)** P(not E and not F).

Ans. Given, $P(E') = \frac{1}{4}$, $P(F') = \frac{1}{2}$, $P(E \cap F') = \frac{1}{8}$

(A) We know: P(E or F) = $P(E \cup F)$

Thus, using, $P(E \cup F) = P(E) + P(F) - P(E \cap F)$

$$= \frac{1}{4} + \frac{1}{2} - \frac{1}{8}$$

$$= \frac{3}{4} - \frac{1}{8}$$

$$= \frac{5}{8}$$

(B) P(not E and not F) = $P(E' \cap F')$

Also, $P(E' \cap F') = P(E \cup F)'$

[From Demorgan's law]

$= 1 - P(E \cup F)$

$$= 1 - \frac{5}{8}$$

$$= \frac{3}{8}$$

Example 1.10: **Case Based:**

Two students Anil and Vijay appeared in an examination. The probability that Anil will qualify the examination is 0.05 and that Vijay will qualify is 0.10. The probability that both will qualify is 0.02.

Based on the above information, answer the following questions.

(A) Assertion (A): The probability that Vijay will not qualify the exam is 0.9.

Reason (R): Probability of happening the event is equal to 1 – probability of not happening.

(a) Both (A) and (R) are true and (R) is the correct explanation of (A).

(b) Both (A) and (R) are true but (R) is not the correct explanation of (A).

(c) (A) is true but (R) is false.

(d) (A) is false but (R) is true.

(B) Find the probability that at least one of them will qualify the exam.

(C) Find the probability that at least one of them will not qualify the exam.

(D) The probability that both Anil and Vijay will not qualify the exam is:

(a) 0.43 (b) 0.67

(c) 0.87 (d) 0.91

(E) The probability that only one of them will qualify the exam is:

(a) 0.21 (b) 0.11

(c) 0.31 (d) 0.25

Ans. Let E_1 and E_2 denotes the events that Anil and Vijay will respectively qualify the exam.

(A) *(a) Both (A) and (R) are true and (R) is the correct explanation of (A).*

Explanation: The probability that Vijay will not qualify the exam

$= 1 - P(E_2)$

$= 1 - 0.10 = 0.9$

(B) $P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2)$

$= 0.05 + 0.10 - 0.02 = 0.13$

(C) Probability of at least one of them does not qualify

$= P(E'_1 \cup E'_2)$

$= P((E_1 \cap E_2)')$

$= 1 - P(E_1 \cap E_2)$

$= 1 - 0.02 = 0.98$

(D) *(c) 0.87*

Explanation: Probability that both Anil and Vijay will not qualify the exam

$= P(E'_1 \cap PE'_2)$

$= P((E_1 \cup E_2)')$

$= 1 - P(E_1 \cup E_2)$

$= 1 - 0.13 = 0.87$

(E) *(b) 0.11*

Explanation: Probability that only one of them will qualify the exam

$= P((E_1 - E_2) \cup (E_2 - E_1))$

$= P(E_1 - E_2) + P(E_2 - E_1)$

$= P(E_1 \cup E_2) - P(E_1 \cap E_2)$

$= 0.13 - 0.02 = 0.11$

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OBJECTIVE Type Questions

[1 mark]

Multiple Choice Questions

1. A box contains 1 red and 3 identical blue balls. Two balls are drawn at random in succession without replacement. Then, the sample space for this experiment is:

(a) {RB,BR,BB} (b) {R, B, B}
(c) {RB} (d) {RB, BR}

Ans. *(a) {RB, BR, BB}*

Explanation:

Let red ball is denoted by R and blue ball is denoted by B. Now, two balls drawn at random in succession without replacement.

Then, the sample space is S = {RB, BR, BB}

2. Find the probability that when a hand of 7 cards are drawn from the well-shuffled deck of 52 cards, it contains all kings.

(a) $\frac{2}{7735}$ (b) $\frac{1}{7735}$
(c) $\frac{3}{7753}$ (d) $\frac{1}{7753}$ [Diksha]

Ans. *(b)* $\frac{1}{7735}$

Explanation: 7 cards are to be chosen from 5C cards. Total cards to be drawn = 7

So, sample space contains = ${}^{52}C_7$

$$P(S) = \frac{52!}{7!(52-7)!} = \frac{52!}{7!45!}$$

There are only 4 kings, so 3 cards come from remaining ones.

So, $P(A) = {}^{48}C_3$

$$= \frac{48!}{3!(48-3)!}$$

$$= \frac{48!}{3!\times 45!}$$

Hence, probability $= \frac{P(A)}{P(S)}$

$$= \frac{48!}{3!\times 45!} \times \frac{7!\times 45!}{52!}$$

$$= \frac{1}{7735}$$

3. A coin is tossed repeatedly until a head comes up for the first time. Then, the sample space for this experiment is:

(a) {H, TH, TTH}
(b) {HH, HHH, TTH, HTT}
(c) {H, TH, TTH, TTTH, TTTTH, ...}
(d) None of these

Ans. *(c) {H, TH, TTH, TTTH, TTTTH, ...}*

Explanation:

The sample space is

S = {H, TH, TTH, TTTH, TTTTH, ...}

4. A pair of dice is rolled, if the outcome is sum 12, a coin is tossed. Then, the total number of outcomes for this experiment is:

(a) 40 (b) 38
(c) 41 (d) 43

Ans. *(b) 38*

Explanation:

The sample space associated with the given random experiment is

S = {(1,1), (1,2), (1,3), (1,4), (1,5), (1,6), (2,1), (2,2), (2,3), (2,4), (2,5), (2,6),
(3,1), (3,2), (3,3), (3,4), (3,5), (3,6), (4,1), (4,2), (4,3), (4,4), (4,5), (4,6), (5,1)
(5,2), (5,3), (5,4), (5,5), (5,6), (6,1), (6,2), (6,3), (6,4), (6,5), (6,6), (6,6,H), (6,6,T)}

Hence, total number of sample points =38

5. A bag contains 4 identical red balls and 3 identical black balls. The experiment consists of drawing one ball, then putting it into the bag and again drawing a ball. Then, the possible outcomes of this experiment is:

(a) {RR, BB} (b) {RR, B, B, RR}
(c) {BB, R} (d) {RR, RB, BR, BB}

Ans. *(d) {RR, RB, BR, BB}*

Explanation: A bag contain 4 red balls and 3 black balls.

Let us assumed Red = R

Let us assume black = B

Now, a ball drawn is first attempt, so elementary event is

$$S_1 = \{R, B\}$$

And, the bag will put into the bag and draw again, then the elementary events are

$$S_2 = \{R, B\}$$

Thus, the total sample space associated is

$$S = S_1S_2$$
$$S = \{RR, RB, BR, BB\}$$

Hence, S is the sample space for the given experiment.

6. A fair coin with 1 marked on one face and 6 on the other and a fair die are both tossed. Find the probability that the sum of numbers that turn up is 3.

(a) $\frac{1}{6}$ (b) $\frac{1}{12}$
(c) $\frac{2}{3}$ (d) $\frac{1}{4}$ [Diksha]

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Ans. *(b) 1/12*

Explanation: S = {(1,1) (1,2) (1,3) (1,4) (1,5) (1,6) (6,1) (6,2) (6,3) (6,4) (6,5) (6,6)}

Total events in sample space is 12

Now probability to come sum as 3 is (1,2) = $\frac{1}{12}$

7. The probability that a non-leap year selected at random will have 52 Sundays is:

(a) 0 (b) 1

(c) $\frac{1}{7}$ (d) $\frac{2}{7}$

[Delhi Gov. QB 2022]

Ans. *(c)* $\frac{1}{7}$

Explanation: A non-leap year has 365 days i.e., 52 weeks and 1 odd days.

So, there are 52 Sundays always.

But the odd day can be Sunday, Monday, Tuesday, Wednesday, Thursday, Friday or Saturday.

So, there are 7 possibilities for odd day and out of which one is favourable.

So, the probability of getting Sunday on last week is $\frac{1}{7}$.

Thus, the probability of getting 53 Sundays in a non-leap year is $\frac{1}{7}$.

8. A couple has two children, find the probability that both children are males, if it is known that at least one of the children is male.

(a) $\frac{2}{3}$ (b) $\frac{1}{3}$

(c) $\frac{4}{5}$ (d) $\frac{5}{3}$ [Diksha]

Ans. *(b)* $\frac{1}{3}$

Explanation: $S = \{MM, MF, FM, FF\}$

Total sample space is 4.

E : Both children are males.

F : atleast one child is male

E = {(M, M)}

F = {(M, F) (F, M) (M, M)}

$$P(E) = \frac{1}{4}$$

$$P(F') = \frac{3}{4}$$

$$P(E \cap F) = \frac{1}{4}$$

$$P(E/F) = \frac{\frac{1}{4}}{\frac{3}{4}} = \frac{1}{3}$$

Assertion Reason Questions

Direction: In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R).

Choose the correct answer out of the following choices.

(a) Both (A) and (R) are true and (R) is the correct explanation of (A).

(b) Both (A) and (R) are true but (R) is not the correct explanation of A.

(c) (A) is true but (R) is false.

(d) (A) is false but (R) is true.

9. Assertion (A): A coin is tossed. If it shows head, we draw a ball from a bag consisting of 3 brown and 4 red balls; if it shows tail we throw a die, then the sample space of this experiment is $S = \{HB_1, HB_2, HB_3, HR_1, HR_2, HR_3, HR_4, T1, T2, T3, T4, T5, T6\}$.

Reason (R): Consider the experiment in which a coin is tossed repeatedly until a head comes up, then the sample space is S = {H, TH, TTH, TTTH, ...}.

Ans. *(b) Both (A) and (R) are true but (R) is not the correct explanation of (A).*

Explanation: Let us denote brown balls by B_1, B_2, B_3 and the red balls by R_1, R_2, R_3, R_4. Then, a sample space of the experiment is $S = \{HB_1, HB_2, HB_3, HR_1, HR_2, HR_3, HR_4, T1, T2, T3, T4, T5, T6\}$.

In the experiment, head may come up on the 1st toss, or the 2nd toss, or the 3rd toss and so on till head is obtained. Hence, the desired sample space is

S = {H, TH, TTH, TTTH, TTTTH, ...}.

10. Assertion (A): A coin is tossed and then a die is rolled only in case a head is shown on the coin. The sample space for the experiment is S = {H1, H2, H3, H4, H5, H6, T}.

Reason (R): 2 boys and 2 girls are in room X, and 1 boy and 3 girls are in room Y. Then, the sample space for the experiment in which a room is selected and then a person, is $S = \{XB_1, XB_2, XG_1, XG_2, YB_3, YG_3, YG_4, YG_5\}$.

where B_i, denote the boys and *Gj*, denote the girls.

Ans. *(b) Both (A) and (R) are true but (R) is not the correct explanation of (A).*

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Explanation: When coin is tossed, either we get head or tail.

Lets head be denoted by H and tail denoted by T.

According to a question when head is shown then a die is rolled.

Hence, Total number of sample space S associated with the experiment.

$$S = \{H_1, H_2, H_3, H_4, H_5, T\}$$

When the room x is selected, then

X	Y
B_1, B_2 G_1, G_2	B_3 G_3, G_4, G_5

There are four possibilities for selection of a person which are B_1, B_2, G_1, G_2, similarly, there will be four possibilities for room Y.

So, the sample space is

$S = \{XB_1, XB_2, XG_1, XG_2, YB_3, YG_3, YG_4, YG_5\}$

11. **Consider the experiment of rolling a die. Then, sample space is S = {1, 2, 3, 4, 5, 6}.**

Assertion (A): **The event E :" the number appears on the die is a multiple of 7", is an impossible event.**

Reason (R): **The event F :" the number turns up is odd or even", is a sure event.**

Ans. *(b) Both (A) and (R) are true but (R) is not the correct explanation of (A).*

Explanation: Given, E be the event "the number appears on the die is a multiple of 7". It is impossible to have a multiple of 7 on the upper face of the die. Thus, the event $E = \phi$ is an impossible event.

The another event F is "the number turns up is odd or even". Clearly, $F = \{1,2,3,4,5,6\} = S$, i.e., all possible outcomes of the experiment ensure the occurrence of the event F. Thus, the event F is a sure event.

12. **Assertion (A):** **If sample space of an experiment is S = {1,2,3,4,5,6} and the events A and B are defined as**

A: "a number less than or equal to 3 appears"

B: "a number greater than or equal to 3 appears"

Then A and B are exhaustive events.

Reason (R): **Events are exhaustive if atleast one of them necessarily occurs whenever the experiment is performed.**

Ans. *(a) Both (A) and (R) are true and (R) is the correct explanation of (A).*

Explanation: If we throw a dice, then the sample space

$$S = \{1, 2, 3, 4, 5, 6\}$$

We have $A = \{1, 2, 3\}$ and $B = \{3, 4, 5, 6\}$

Since, $A \cup B = S$, so, A and B are exhaustive events.

13. **Assertion (A):** **The probability of drawing either an ace or a king from a pack of cards in a single draw is $\frac{2}{13}$.**

Reason (R): **For two events A and B which are not mutually exclusive, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.**

Ans. *(b) Both (A) and (R) are true but (R) is not the correct explanation of (A).*

Explanation: P (Drawing either an ace or a king)

$$= \frac{4}{52} + \frac{4}{52} = \frac{8}{52}$$

$$= \frac{2}{13}$$

($\because$ Both events are mutually exclusive)

If two events A and B are not mutually exclusive, then $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

CASE BASED Questions (CBQs)

[**4** & **5** marks]

Read the following passages and answer the questions that follow:

14. *One card is drawn from a well shuffled deck of 52 cards. Each outcome is equally likely.*

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(A) The probability that the card will be a heart is:

(a) $\frac{1}{4}$ (b) $\frac{1}{2}$

(c) $\frac{3}{4}$ (d) $\frac{1}{52}$

(B) The probability that the card will be a black card is:

(a) $\frac{1}{4}$ (b) $\frac{1}{2}$

(c) $\frac{3}{4}$ (d) $\frac{1}{52}$

(C) The probability that the card will be an ace of spade is:

(a) $\frac{1}{4}$ (b) $\frac{1}{2}$

(c) $\frac{3}{4}$ (d) $\frac{1}{52}$

(D) The probability that the card will be a king of red color is:

(a) $\frac{1}{52}$ (b) $\frac{1}{26}$

(c) $\frac{1}{13}$ (d) $\frac{1}{39}$

(E) The probability that the card will be a face card is:

(a) $\frac{3}{13}$ (b) $\frac{4}{13}$

(c) $\frac{2}{13}$ (d) none of these

Ans. (A) *(a)* $\frac{1}{4}$

Explanation: Total number of possible outcomes = 52

Probability of drawing a heart card = $\frac{13}{52} = \frac{1}{4}$

(B) *(b)* $\frac{1}{2}$

Explanation: Probability of drawing a black card = $\frac{26}{52} = \frac{1}{2}$

(C) *(d)* $\frac{1}{52}$

Explanation: Probability of drawing an ace of spade = $\frac{1}{52}$.

(D) *(b)* $\frac{1}{26}$

Explanation: Probability of drawing a king of red color = $\frac{2}{52} = \frac{1}{26}$

(E) *(a)* $\frac{3}{13}$

Explanation: Cards of king, queen and jack are called face cards and there are four suits, namely, heart, diamond, club and spade.

Probability of drawing a face card = $\frac{12}{52} = \frac{3}{13}$

15. *During the COVID-19 pandemic due to shortage of doctors team of medical students doing their intership were asked to assist senior doctors during surgeries at city hospital. The probabilities of surgeries rated as very complex, complex routine simple or very simple are respectively. As shown below:*

Very Complex	Complex	Routine	Simple	Very Simple
0.15	0.20	0.31	0.26	0.08

Based on the above information answer the following questions.

Find the probability that particular surgery will be:

(A) Complex or very complex

(B) Neither very complex nor very simple.

(C) If P(A) is $\frac{3}{5}$. Find P(not A).

[Delhi Gov. Term-2 SQP 2022]

Ans. (A) P(complex or very complex)

$= P(E_1 \text{ or } E_2)$
$= P(E_1 \cup E_2)$
$= P(E_1) + P(E_2) - P(E_1 \cap E_2)$
$= 0.15 + 0.20 - 0 = 0.35$

(B) P(neither very complex nor very simple)

$= P(E'_1 \cap E'_5)$
$= P(E_1 \cup E_5)'$
$= 1 - P(E_1 \cup E_5)$
$= 1 - [P(E_1) + P(E_5)]$
$= 1 - (0.15 + 0.08)$
$= 1 - 0.23$
$= 0.77$

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(C) Given that, $P(A) = \frac{3}{5}$

To find $P(\text{not } A) = 1 - P(A)$

$$P(\text{not } A) = 1 - \frac{3}{5}$$

$$= \frac{(5-3)}{5}$$

$$= \frac{2}{5}$$

Therefore, $P(\text{not } A) = \frac{2}{5}$.

16. *On a Diwali's night, four members of a family plan to play a Ludo game. Head of the family is act as an initiator of the game. He throws a coin and a die.*

(A) Find the number of sample points and probability of getting a head.

(B) Find the probability of getting and odd number and also find the probability of getting a head and an even number.

(C) One die of red colour, one of white colour and one of blue colour are placed in a bag. One die is selected at random and rolled, its colour and the number on its uppermost face is noted. Describe the sample space.

Ans. (A) When a coin and a die are thrown, the sample space, *S*, is given by

S = {H1, H2, H3, H4, H5, H6, T1, T2, T3, T4, T5, T6}

The number of sample points is 12.

Probability of getting a head = $\frac{6}{12} = \frac{1}{2}$

(B) Probability of getting an odd number $= \frac{6}{12} = \frac{1}{2}$

Probability of getting a head and an even number = $\frac{3}{12} = \frac{1}{4}$

(C) Let us assume that 1, 2, 3, 4, 5 and 6 are the possible numbers that come when the die is thrown.

And also, assume die of red colour be '*R*', die of white colour be '*W*', die of blue colour be '*B*'.

So, the total number of sample space $= (6 \times 3) = 18$

The sample space of the event is

S = {(*R*, 1), (*R*, 2), (*R*, 3), (*R*, 4), (*R*, 5), (*R*, 6), (*W*, 1), (*W*, 2), (*W*, 3), (*W*, 4), (*W*, 5), (*W*, 6), (*B*, 1), (*B*, 2), (*B*, 3), (*B*, 4), (*B*, 5), (*B*, 6)}

VERY SHORT ANSWER Type Questions (VSA)

[1 mark]

17. One number is chosen at random from the numbers 1 to 21. What is the probability that it is prime? [Delhi Gov. QB 2022]

Ans. Sample space $n(S) = 21$

Prime numbers from 1 to 21 are 2, 3, 5, 7, 11, 13, 17, 19.

It 'E' be the event of getting a prime number, then $n(E) = 8$

$$\therefore \quad p(E) = \frac{n(F)}{n(S)} = \frac{8}{21}$$

∴ The probability that the number's prime $= \frac{8}{21}$.

18. Events *E* and *F* are such that *P*(not *E* or not *F*) = 0.25, state whether E and F are mutually exclusive. **[Diksha]**

Ans. From the question it is given that, P(not E and not F) = 0.25

So, $P(\bar{E} \cup \bar{F}) = 0.25$

Then we have,

$\Rightarrow \quad P(\bar{E} \cup \bar{F}) = 0.25$

$\Rightarrow \quad 1 - P(E \cap F) = 0.25$

$\Rightarrow \quad P(E \cap F) = 1 - 0.25$

$= 0.75$

$P(E \cap F) \neq 0$

Hence, *E* and *F* are not mutually exclusive events.

19. Three identical dice are rolled. Find the probability that the same number appears on each of them. [Delhi Gov. QB 2022]

Ans. 3 dices are thrown $n = 6^3$.

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Same number appears mean sample space is

A = {(1, 1, 1), (2, 2, 2), (3, 3, 3), (4, 4, 4), (5, 5, 5), (6, 6, 6)}

$$\therefore \quad P(A) = \left(\frac{6}{6^3}\right) = \left(\frac{1}{36}\right)$$

20. 2 boys and 2 girls are in room A and 1 boy and 2 girls are in room B. Find the sample space in which room is selected and then a person.

Ans. Let B_1, B_2, G_1, G_2 are in room A and B_3, G_3, G_4 are in room B, then

$S = \{AB_1, AB_2, AG_1, AG_2, BB_3, BG_3, BG_4\}$ are the sample space.

21. A coin is tossed four times. Find the sample space. [Diksha]

Ans. Since either coin can turn up Head (H) or Tail (T), are the possible outcomes.

So, when 1 coin is tossed once the sample space

= 2

Then,

Coin is tossed 4 times the sample space $= 2^4 = 16$

Thus, the sample space is S = {$HHHH$, $THHH$, $HTHH$, $HHTH$, $HHHT$, $TTTT$, $HTTTT$, $THTT$, $TTHT$, $TTTH$, $TTHH$, $HHTT$, $THTH$, $HTHT$, $THHT$, $HTTH$}

22. If the letters of the word 'ALGORITHM' are arranged at random in a row, what is the probability the letter 'GOR' must remain together as a unit? [NCERT Exemplar]

Ans. Number of letters in the word 'ALGORITHM' are 9 if 'GOR' remain together, then considered it as 1 number.

$\therefore$ Number of letters = 6 + 1 = 7

Number of word, if 'GOR' remain together = 7!

Total number of words from the letters of the word 'ALGORITHM' = 9!

$\therefore$ Required probability

$$= \frac{7!}{9!} = \frac{7!}{9 \times 8 \times 7!} = \frac{1}{9 \times 8} = \frac{1}{72}$$

SHORT ANSWER Type-I Questions (SA-I)

[2 marks]

23. A die is loaded in such a way that each odd number is twice as likely to occur as each even number. Find $P(G)$, where G is the event that a number greater than 3 occurs on a single roll of the die. [NCERT Exemplar]

Ans. It is given that, 2× Probability of even number = Probability of odd number

$\Rightarrow \quad P(A) = 2P(B)$

Let (A odd number, B even number)

$\Rightarrow \quad P(A) : P(B) = 2 : 1$

$\therefore$ Probability of occuring, odd number,

$$P(A) = \frac{2}{2+1} = \frac{2}{3}$$

And probability of occurring even number,

$$P(B) = \frac{2}{2+1} = \frac{1}{3}$$

Now, G be the even that a number greater than 3 occur in a single roll of die.

So, the possible outcomes are 4, 5 and 6 out of which two are even and one odd.

$\therefore$ Required probability = P(G) = 2 × P(A) × P(B)

$$= 2 \times \frac{2}{3} \times \frac{1}{3} = \frac{4}{9}$$

24. Punching time of an employee is given below:

Day	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
Time	10:35	10:20	10:22	10:27	10:25	10:40

If the reporting time is 10:30 a.m., then find the probability of his coming late. [Delhi Gov. QB 2022]

Ans. Sample space n(S) = 6

If the reporting time is 10:30 a.m. then the event is given by

$$n(L) = 2$$

$$\therefore \quad \text{Required probability} = \frac{n(L)}{n(S)}$$

$$= \frac{2}{6}$$

$$= \frac{1}{3}$$

25. If E and F are events such that P(E) = $\frac{1}{4}$, P(F) = $\frac{1}{2}$ and P(E and F) = $\frac{1}{8}$, find

(A) P(E or F), (B) P(not E and not F)

[Diksha]

Ans. From the question, we have P(E) = $\frac{1}{4}$, P(F) = $\frac{1}{2}$ and P(E and F) = $\frac{1}{8}$ and

(A) We know that P(E or F)

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i.e., $P(E \cup F) = P(E) + P(F) - P(E \cap F)$

$$= \frac{1}{4} + \frac{1}{2} - \left(\frac{1}{8}\right)$$

$$= \frac{2+4-1}{8}$$

$$= \frac{5}{8}$$

(B) P(not E and not F) = $P(\bar{E} \cap \bar{F}) = 1 - P(E \cup F)$

$$= 1 - \left(\frac{5}{8}\right)$$

$$= \frac{(8-5)}{8}$$

$$= \frac{3}{8}$$

26. Three balls are drawn from a bag containing 5 red, 4 white, and 3 black balls. Find the number of ways in which this can be done if atleast 2 balls are red. [Delhi Gov. SQP 2022]

Ans. Number of red balls = 5

Number of white balls = 4

Number of black balls = 3

Number of ball drawn = 3

Note, at least 2 red balls can be drawn in following ways:

⇒ 2 red and 1 non red.

⇒ all 3 reds balls.

∴ Number of ways of drawing atleast two red balls is all red ${}^5C_3 + {}^5C_2 \times {}^7C_1$

$$= \frac{4 \times 5}{2} + \frac{4 \times 5}{2} \times 7$$

$$= 10 + 35 \times 2$$

$$= 80$$

27. Find P (not A and not B), If A and B are two events such that $P(A) = \frac{1}{6}$, $P(B) = \frac{1}{2}$ and $P(A \cap B) = \frac{1}{12}$.

Ans. P (not A and not B) = $P(\bar{A} \cap \bar{B})$

$$= 1 - P(A \cup B)'$$

$$= 1 - P(A) - P(B) + P(A \cap B)$$

$$= 1 - \frac{1}{6} - \frac{1}{2} + \frac{1}{12}$$

$$= \frac{5}{12}$$

28. In a large metropolitan area, the probabilities are 0.87,0.36,0.30 that a family (randomly chosen for a sample survey) owns a colour television set, a black and white television set or both kinds of sets. What is the probability that a family owns either anyone or both kinds of sets? [NCERT Exemplar]

Ans. Let E_1 be the event that family own colour television set and E_2 be the event that family own Black and White television.

It is given that

$$P(E_1) = 0.87$$

$$P(E_2) = 0.36$$

And $P(E_1 \cap E_2) = 0.30$

We have to find probability that a family owns either anyone or both kind of sets i.e., $p(E_1 \cup E_2)$.

Now, $P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2)$

[by addition theorem]

$$= 0.87 + 0.36 - 0.30$$

$$= 0.93$$

29. A coin is tossed and then a die is rolled only in case a head is shown on the coin. Find the total sample space when tail is encountered. [Diksha]

Ans. Since either coin can turn up Head (H) or Tail (T), are the possible outcomes.

Let us assume that 1,2,3,4,5 and 6 are the possible numbers that come when the die is thrown.

When head in encountered,

Then, number of space = (1 × 6) = 6

Sample space S_H = {H1, H2, H3, H4, H5, H6}

Now, tail is encountered, sample space S_T = {T}

Therefore the total sample space S = {H1, H2, H3, H4, H5, H6, T}

SHORT ANSWER Type-II Questions (SA-II)

[3 marks]

30. Three candidates A, B and C are going to play in a chess competition to win FIDE (World Chess Federation) cup this year. A is thrice is likely to win as B and B is twice as likely to win C. Find the respective probability of A, B and C to win the cup. [Delhi Gov. QB 2022]

Ans. Let E_1 be the event of winning of A.

E_2 be the event of winning of B.

E_3 be the event of winning of C.

According to the question,

$$E_1 : E_2 = 3 : 1$$

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$$E_2 : E_3 = 2 : 1$$
$$E_1 : E_2 : E_3 = 6 : 2 : 1$$
Let
$$E_1 = 6k$$
$$E_2 = 2k$$
$$E_3 = \frac{k}{9k}$$
$$P(E_1) = \frac{6k}{8k} = \frac{2}{3}$$
$$P(E_2) = \frac{2k}{9k} = \frac{2}{9}$$

31. Six new employees, two to whom are married to each other, are to be assigned six desks that are lined up in a row. If the assignment of employees to desks is made randomly, what is the probability that the married couple will have non-adjacent desks? [NCERT Exemplar]

Ans. Let the couple occupied adjacent desks, consider those two as 1.

There are, (4 + 1) i.e., 5 persons to be assigned.

∴ Number of ways of assigning these five persons = 5! × 2!

Total number of ways of assigning 6 persons = 6!

∴ Probability that the couple has adjacent desk $= \frac{5! \times 2!}{6!} = \frac{1}{3}$

Probability that the married couple will have non-adjacent desks $= 1 - \frac{1}{3} = \frac{2}{3}$

32. One card is drawn from a pack of 52 cards. Find the probability that the card is:

(A) a face cards. (B) not a diamond.

(C) an ace.

Ans. Total number of cards =52

Number of cards drawn = 1

(A) Number of favourable cases that the card is a face card =12.

Hence, required probability $= \frac{12}{52} = \frac{3}{13}$.

(B) Number of favourable cases that the card is not a diamond = 39.

Hence, required probability $= \frac{39}{52} = \frac{3}{4}$.

(C) Number of favourable cases that the card is an ace =4.

Hence, required probability $= \frac{4}{52} = \frac{1}{13}$.

33. All the face cards are removed from a deck of 52 playing cards. The remaining cards are well shuffled and then one card is drawn at random. Giving ace a value 1 and similar value for other cards. Find the probability of getting a card with value less than 7.

[Delhi Gov. QB 2022]

Ans. In out of 52 playing cards, 4 jacks, 4 queens and 4 kings are removed, then the remaining cards are left, $n(S) = 52 - 3 \times 4 = 40$.

Let E = Event of getting a card whose value is less than 7.

= Event of getting a card whose value is 1, 2, 3, 4, 5 or 6

$$n(E) = 6 \times 4 = 24$$

$$\therefore \quad P(E_3) = \frac{n(E_3)}{n(S)} = \frac{24}{40} = \frac{3}{5}$$

34. There are four men and six women on the city council. If one council member is selected for a committee at random, how likely is it that she is a woman? [Diksha]

Ans. From the question, it is given that there are four men and six women on the city council.

Here, the total members in the council = 4 + 6 = 10

Hence, the sample space has 10 points

∴ $n(S) = 10$

Number of women are 6. ... (given)

Let us assume 'A' be the event of selecting a woman.

Then $n(A) = 6$

$$P(\text{Event}) = \frac{\text{Number of outcomes favourable to event}}{\text{Total number of possible outcomes}}$$

$$\therefore \quad P(A) = \frac{n(A)}{n(S)} = \frac{6}{10} = \frac{3}{5}$$

35. An experiment consists of a rolling a die until a 2 appears.

(A) How many elements of the sample space correspond to the event that the 2 appears on the k^{th} roll of the die?

(B) How many elements of the sample space correspond to the event that the 2 appears not later than the k^{th} roll of the die? [NCERT Exemplar]

Ans. In a throw of a die there are 6 sample points.

(A) If 2 appears on the k^{th} roll of the die.

So, first $(k - 1)$ roll have 5 outcomes each and k^{th} roll results 2 i.e., 1 outcomes.

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∴ Number of element of sample space correspond to the die, then 2 appears on the k^{th} roll of the die = 5^{k-1}.

$k-1$

(B) If we consider that 2 appears not later than k^{th} roll of the die, then it is possible that 2 comes in first throw, i.e., 1 outcome.

36. **A bag contains 8 red and 5 white balls. Three balls are drawn at random. Find the probability that**

(A) all the three balls are white.

(B) all the three balls are red.

(C) one ball is red and two balls are white.

[NCERT Exemplar]

Ans. Total number of balls = 8 + 5 = 13

Total number of events for drawing 3 balls $= {}^{13}C_3$

(A) Total number of events for getting white balls $= {}^{5}C_3$

$$P(\text{all 3 balls white}) = \frac{{}^{5}C_3}{{}^{13}C_3}$$

$$= \frac{5!}{3! \times 2!} \times \frac{3! \times 10!}{13!}$$

$$\Rightarrow \quad P(\text{all 3 balls white}) = \frac{5}{143}$$

(B) Favourable number of events for getting red balls $= {}^{8}C_3$

P(all 3 balls red)

$$= \frac{{}^{8}C_3}{{}^{13}C_3}$$

$$= \frac{8!}{3! \times 5!} \times \frac{3! \times 10!}{13!}$$

$$\Rightarrow \quad P(\text{all 3 balls red}) = \frac{28}{143}$$

(C) Favourable number of events for getting 1 red ball $= {}^{8}C_1$

Favourable number of events for getting 2 white balls $= {}^{5}C_2$

P(1 red and 2 white balls)

$$= \frac{{}^{8}C_1 \times {}^{5}C_2}{{}^{13}C_3}$$

$$= \frac{8!}{1! \times 7!} \times \frac{5!}{2! \times 3!} \times \frac{3! \times 10!}{13!}$$

$$\Rightarrow \quad P(\text{all 3 balls red}) = \frac{40}{143}$$

LONG ANSWER Type Questions (LA)

[4 & 5 marks]

37. **An urn contains 5 blue and an unknown number x of red balls. Two balls are drawn at random. If the probability of both of them being blue is $\frac{5}{14}$, find x. [Delhi Gov. QB 2022]**

Ans. Number of blue balls = 5

Let the number of red balls be x

Total number of balls = 5 + x

The probability of drawing two blue balls = $\frac{5}{14}$

$$\Rightarrow \quad \frac{5}{14} = \frac{{}^{5}C_2}{{}^{5+x}C_2} \quad \text{...(i)}$$

We know that,

$${}^{n}C_r = \frac{n!}{r!(n-r)!}$$

$${}^{5}C_2 = \frac{5!}{2!3!}$$

$$= \frac{5 \times 4 \times 3!}{2 \times 3!}$$

$$= \frac{5 \times 4}{2} = 10$$

$${}^{5+x}C_2 = \frac{(5+x)!}{2!(3+x)!}$$

$$= \frac{(5+x)(4+x)(3+x)!}{2(3+x)!}$$

$$= \frac{(5+x)(4+x)}{2}$$

Substitute the values in equation (i)

$$\frac{5}{14} = \frac{10 \times 2}{(5+x)(4+x)}$$

$$\Rightarrow \quad 56 = (5+x)(4+x)$$

$$\Rightarrow \quad 56 = x^2 + 9x + 20$$

$$\Rightarrow \quad 0 = x^2 + 9x - 36$$

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$\Rightarrow \quad 0 = x^2 + 12x - 3x - 36$

$\Rightarrow \quad 0 = x(x + 12) - 3(x + 12)$

$\Rightarrow \quad 0 = (x + 12)(x - 3)$

On solving this quadratic equation, we get

$x = -12$ or $x = 3$

x cannot be negative. So, $x = 3$.

38. If an integer from 1 through 1000 is chosen at random, then find the probability that the integer is a multiple of 2 or a multiple of 9.

Ans. Multiple of 2 from 1 to 1000 are 2, 4, 6, 8, ..., 1000.

Let n be the number of terms of above series 1 to 1000.

$\therefore \quad l = 1000$

$\Rightarrow \quad 2 + (n - 1)2 = 1000$

$\{l = a + (n - 1)d, a = 2, d = 2\}$

$\Rightarrow \quad 2(n - 1) = 998$

$\Rightarrow \quad n - 1 = 499$

$\Rightarrow \quad n = 500$

$$p(A) = \frac{500}{1000}$$

$l = 999$

$a = 9, d = 9$

These are m numbers.

Since, the number of multiples of 2 are 500.

So, the multiple of 9 are 9,18,27, ..., 999

$\therefore \quad 9 + (m - 1) = 999$

$\Rightarrow \quad 9(m - 1) = 990$

$\Rightarrow \quad m - 1 = 110$

$\therefore \quad m = 111$

$$P(B) = \frac{111}{1000}$$

Since, the number of multiple of 9 are 111, now the multiple of 2 and 9 both are 18,36, ..., 990

Let p be the number of terms in the above series.

$\therefore \quad p^{th}$ term $= 990$

$\Rightarrow \quad 18 + (p - 1)\,18 = 990$

$\Rightarrow \quad (p - 1)\,18 = 972$

$\Rightarrow \quad p - 1 = 54$

$\Rightarrow \quad p = 55$

and $\quad p(A \cap B) = \dfrac{55}{1000}$

Required probability

$= p(A) + p(B) - p(A \cap B)$

$$\frac{500+111-55}{1000} = \frac{556}{1000} = .556$$

39. Three coins are tossed once. Find the probability of getting

(A) 3 heads **(B) 2 heads**

(C) at least 2 heads **(D) at most 2 heads**

(E) no head **[Diksha]**

Ans. Since, either coin can turn up Head (H) or Tail (T), are the possible outcomes.

But, now three coin is tossed so the possible sample space contains,

S = {HHH, HHT, HTH, THH, TTH, HTT, TTT, THT}

Where s is sample space and here $n(S) = 8$

(A) 3 heads

Let us assume 'A' be the event of getting 3 heads

$n(A) = 1$

$$\therefore \quad P(A) = \frac{n(A)}{n(S)} = \frac{1}{8}$$

(B) 2 heads

Let us assume 'B' be the event of getting 2 heads

$n(A) = 3$

$$\therefore \quad P(B) = \frac{n(B)}{n(S)} = \frac{3}{8}$$

(C) at least 2 heads

Let us assume 'C' be the event of getting at least 2 head

$n(C) = 4$

$$\therefore \quad P(B) = \frac{n(C)}{n(S)} = \frac{4}{8} = \frac{1}{2}$$

(D) at most 2 heads

Let us assume 'D' be the event of getting at most 2 heads

$n(D) = 7$

$$\therefore \quad P(D) = \frac{n(D)}{n(S)} = \frac{7}{8}$$

(E) no head

Let us assume 'E' be the event of getting no heads

$n(E) = 1$

$$\therefore \quad P(E) = \frac{n(E)}{n(S)} = \frac{1}{8}$$

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40. If the length of the 'ASSASSINATION' are arranged at random. Find the probability that
(A) four S's come consecutively in the word.
(B) two I's and two N's come together.
(C) all A's are not coming together.
(D) no two A's are coming together.
[NCERT Exemplar]

Ans. Total number of letters in the word 'ASSASSINATION' are 13.
Out of which 3A'S, 4S's, 2I's, 1T's and 10.

(A) If four S's come consecutively in the word, then we considered these 4S's as 1 group
Now, the number of letters is 10.

S	S	S	S	A	A	A	I	I	N	N	T	O
1				9								

Number of words all S's are together

$$= \frac{10!}{3!2!2!}$$

Total number of words using letters of the word 'ASSASSINATION'

$$= \frac{13!}{3!4!2!2!}$$

$$\therefore \text{Required probability} = \frac{10!}{3!2!2!} \times \frac{3!4!2!2!}{13!}$$

$$= \frac{10! \times 4!}{13!} = \frac{4!}{13 \times 12 \times 11}$$

$$= \frac{24}{1716} = \frac{2}{143}$$

(B) If 2I's and 2N's come together, then there as 10 alphabets.
Number of word when 2I's and 2N's are come together.

$$= \frac{10!}{3!4!} \times \frac{4!}{2!2!}$$

$$\therefore \text{Required probability} = \frac{\frac{10!4!}{3!4!2!2!}}{\frac{13!}{3!4!2!2!}}$$

$$= \frac{4!10!}{2!2!3!4!} \times \frac{3!4!2!2!}{13!}$$

$$= \frac{4!10!}{13!} = \frac{4!}{13 \times 12 \times 11}$$

$$= \frac{24}{13 \times 12 \times 11} = \frac{2}{143}$$

(C) If all A's are coming together, then there are 11 alphabets.
Number of words when all A's come together

$$= \frac{11!}{4!2!2!}$$

Probability when all A's come together

$$= \frac{\frac{11!}{4!2!2!}}{\frac{13!}{4!3!2!2!}} = \frac{11!}{4!2!2!} \times \frac{4!3!2!2!}{13!}$$

$$= \frac{11! \times 3!}{13!} = \frac{6}{13 \times 12} = \frac{1}{26}$$

Required probability when all A's does not come together

$$= 1 - \frac{1}{26} = \frac{25}{26}$$

(D) If no two A's are together, then first we arrange the alphabets except A's.

	S		S		S		S		I		N		T		I		O		N	

All the alphabets except A's are arranged in $\frac{10!}{4!2!2!}$.

There are 11 vacant places between these alphabets.

So, 3A's can be place in 11 places in $^{11}C_3$ ways $= \frac{11!}{3!8!}$

$\therefore$ Total number of words when no two A's are together

$$= \frac{11!}{3!8!} \times \frac{10!}{4!2!2!}$$

$$\text{Required probability} = \frac{11! \times 10!}{3!8!4!2!} \times \frac{4!3!2!2!}{13!}$$

$$= \frac{10!}{8! \times 13 \times 12}$$

$$= \frac{10 \times 9}{13 \times 12} = \frac{90}{156} = \frac{15}{26}$$

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